Multiphase Flow and Heat Transfer

Boiling

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Basics

Phase

Macroscopic state of matter which is homogeneous in chemical composition and physical structure. Gas, Liquid & Solid.

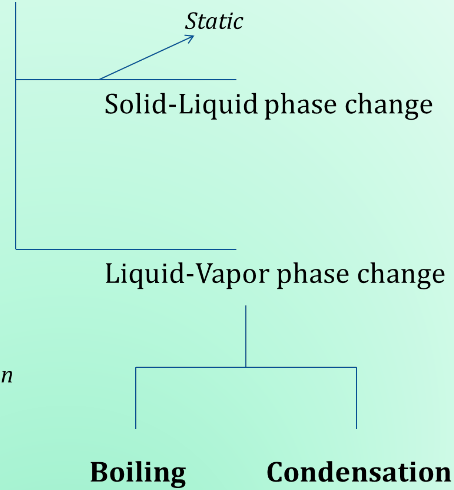
Gas-Liquid
  Steam and water; Air and water

Liquid-Solid
  Plasma and platelets (Blood)

Liquid-Liquid
  Oil and water

Phase Change

Melting and Solidification → No motion



Static

Solid-Liquid phase change

Liquid-Vapor phase change

Continuous, also energy equation

Boiling

Condensation

Boiling and Condensation

Thermodynamics

- When the temperature of a liquid at a specified pressure is raised to the saturation temperature T_{sat} at that pressure, Boiling occurs.
- Liquid-Vapor transformation: $T_s > T_{sat}$ at a given pressure.

- When the temperature of a vapor is lowered to T_{sat}, Condensation occurs.
- Vapor-Liquid transformation: $T_s < T_{sat}$ at a given pressure.

Typical Values of Heat Transfer Coefficient

$$q_{conv} = hA_s \left(T_s - T_\infty\right)$$

Process	**h (W/m^2 K)**
Free convection	
Gases	2-25
Liquids	50-1,000
Convection with phase change	
Boiling and Condensation	2,500-100,000

Physical Considerations

Free and Forced convection depends on

$\rho, C_p, \mu, k_{fluid}$

Boiling/Condensation Heat Transfer depends on

- $\rho, C_p, \mu, k_{fluid}$
- $\Delta T = |T_s - T_{sat}|$
- Latent heat of vaporization, h_{lv}
- Surface tension at the liquid-vapor interface, σ
- body force arising from the liquid-vapor density difference, $g(\rho_l - \rho_v)$

$$h = h[\Delta T, g(\rho_l - \rho_v), h_{lv}, \sigma, L, \rho, C_p, k, \mu]$$

10 variables in 5 dimensions $\implies$ 5 pi-groups.

Dimensionless Parameters

$$\frac{hL}{k} = f\left[\frac{\rho g(\rho_l - \rho_v)L^3}{\mu^2}, \frac{C_p \Delta T}{h_{lv}}, \frac{\mu C_p}{k}, \frac{g(\rho_l - \rho_v)L^2}{\sigma}\right]$$

$$\mathrm{Nu}_L = f\left[\frac{\rho g(\rho_l - \rho_v)L^3}{\mu^2}, \mathrm{Ja}, \mathrm{Pr}, \mathrm{Bo}\right]$$

Jakob number

Ratio of max sensible energy absorbed by liquid (vapor) to latent energy absorbed by liquid (vapor) during boiling (condensation).

Bond number

Ratio of the buoyancy force to the surface tension force.

Unnamed parameter

Represents the effect of buoyancy-induced fluid motion on heat transfer.

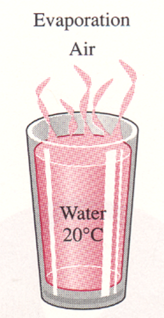
Boiling and Evaporation

Boiling

- The process of addition of heat to a liquid such a way that generation of vapor occurs.
- Solid-liquid interface
- Characterized by the rapid formation of vapor bubbles

Evaporation

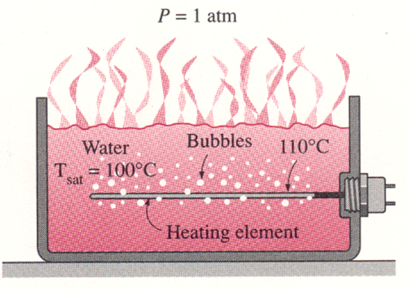
- Liquid-vapor interface
- $P_v < P_{sat}$ of the liquid at a given temp
- No bubble formation or bubble motion



Boiling

Boiling occurs

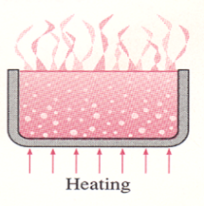
- Solid-liquid interface
- when a liquid is brought into contact with a surface at a temperature above the saturation temperature of the liquid

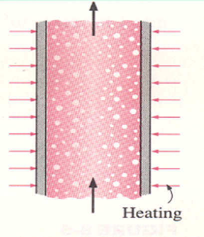


Bubble

- The boiling processes in practice do not occur under equilibrium conditions.
- Bubbles exist because of the surface tension at the liquid vapor interface due to the attraction force on molecules at the interface toward the liquid phase.
- The temperature and pressure of the vapor in a bubble are usually different than those of the liquid.
- Surface tension $\downarrow$ $\uparrow$ Temperature
- Surface tension $= 0$ at critical temperature
- No bubbles at supercritical pressures and temperatures

Boiling - Classification





Pool boiling

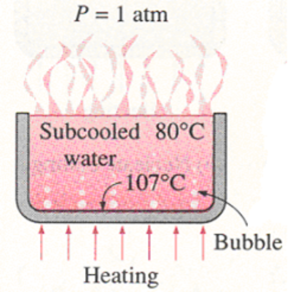
- Fluid is stationary
- Fluid motion is due to natural convection currents
- Motion of bubbles under the influence of buoyancy

Flow boiling

- Fluid is forced to move in a heated pipe or surface by external means such as pump
- Always accompanied by other convection effects

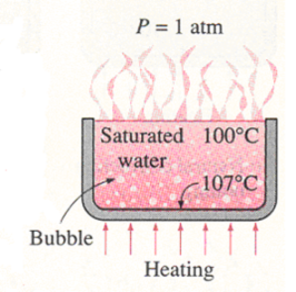
Boiling - Classification

Subcooled boiling



$T_{\text{bulk of liquid}} < T_{sat}$

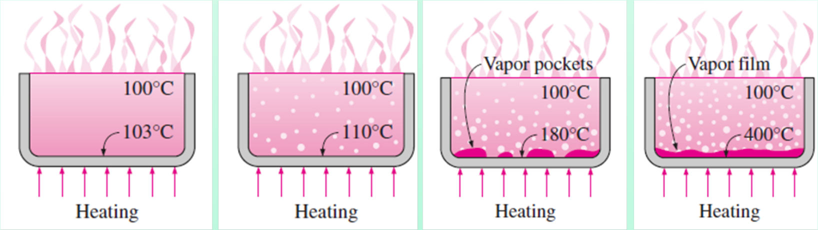
Saturated boiling



$T_{\text{bulk of liquid}} = T_{sat}$

Boiling Regimes

Boiling curve for saturated water at atmospheric pressure

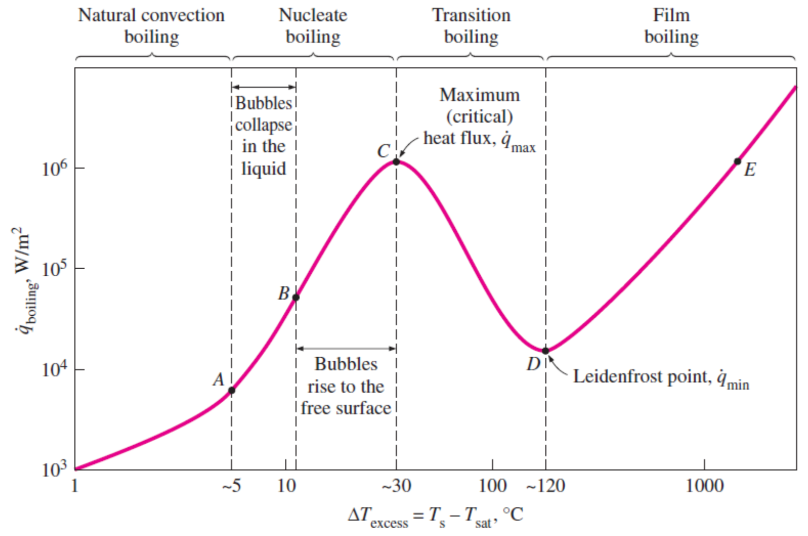


Natural convection boiling **Nucleate boiling** **Transition boiling** **Film boiling**

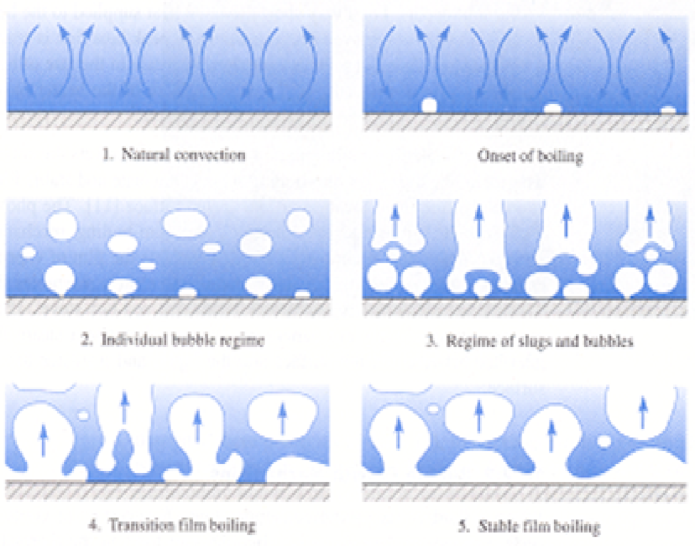
Nichrome (1500 K), Platinum (2045 K)

Nukiyama, 1934: Nichrome, Platinum

Boiling curve for saturated water at atmospheric pressure



Boiling Regimes - Nukiyama, 1934



1. Natural convection

Onset of boiling

2. Individual bubble regime

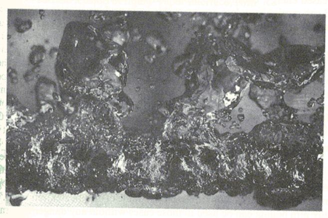
3. Regime of slugs and bubbles

4. Transition film boiling

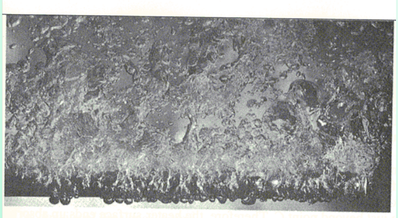
5. Stable film boiling

Boiling Regimes

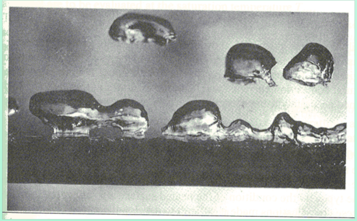
Methanol on horizontal 1 cm steam-heated copper tube



Nucleate boiling

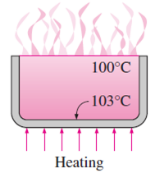


Transition boiling



Film boiling

Boiling Regimes

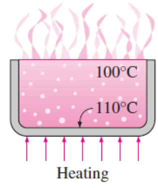


Natural convection

- Governed by natural convection currents.
- Heat transfer from the heating surface to the fluid is by natural convection.
- Natural convection ends at $\Delta T = 5°$C.

Nucleate boiling

- Onset of nucleate boiling (ONB)
- Stirring and agitation caused by the entrainment of the liquid to the heater surface increases h, q''.
- High heat transfer rates are achieved.

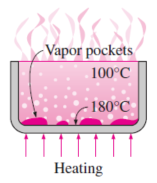


Boiling Regimes

Nucleate boiling

- $A - B$, isolated bubbles are formed at various preferential nucleation sites on the heated surface
- Bubbles collapse in the liquid.
- $B - C$, bubbles form at great rates forming continuous columns of vapor.
- Move all the way up to the free surface, where they break up and release their vapor content.
- **Critical/maximum heat flux (CHF)**, $\dot{q}''_{max}$
- For water, CHF = 1 MW/m^2 at 30 K excess temperature.
- $h = \dot{q}''_{max}/\Delta T_{sat} = 3.3 \times 10^4$ W/m^2 K.

Boiling Regimes

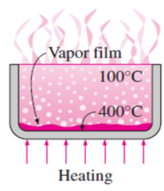


Transition boiling

- Large fraction of the heater surface is covered by a vapor film.
- Both nucleate and film boiling partially occur.
- Unstable film boiling regime.
- Avoided in practice.

Film boiling

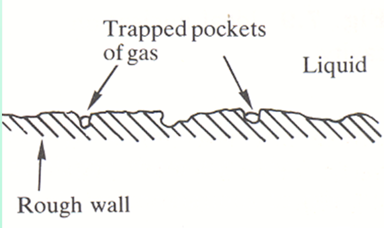
- Vapor film is responsible for the low heat transfer
- $\dot{q}$ increases with increasing ΔT as a result of heat transfer from the heated surface to the liquid through the vapor film by radiation.



Boiling Nucleation - Boiling Inception

The process of bubble formation is called Nucleation

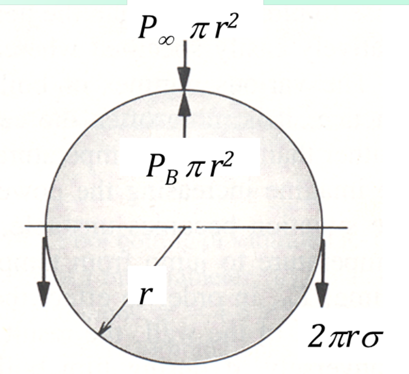
Enlarged view of a boiling surface



- The cracks and crevices do not constitute nucleation sites for the bubbles. Must contain pockets of gas/air trapped
- It is from these pockets of trapped air that the vapor bubbles begin to grow during nucleate boiling
- These cavities are the sites at which bubble nucleation occurs

Boiling Nucleation - Boiling Inception

When a liquid contacts the surface, surface tension forces prevent the liquid from entering the smaller cavities in which air or other gases are trapped.



r	radius of the bubble
σ	surface tension
P_B	pressure inside the bubble
P_∞	pressure in the liquid or the ambient pressure

Boiling Nucleation - Boiling Inception

For static equilibrium, the surface tension force balances the net pressure force:

$$2\pi r\sigma = (P_B - P_\infty)\,\pi r^2$$

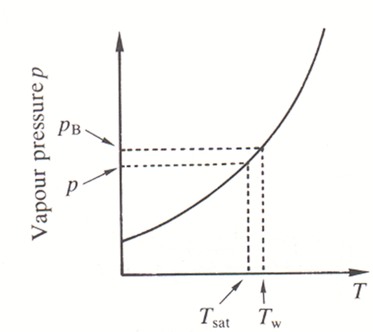
$$\boxed{\boldsymbol{P_B = P_\infty + \frac{2\sigma}{r}}}$$

Young-Laplace equation

P_B is maximum when $r = R$ (the cavity radius)

The wall temperature T_w must be high enough to vaporize the liquid at a pressure of P_B.

Boiling Nucleation - Boiling Inception



Vapor pressure curve: superheat required for nucleation

For the bubble to grow, the required condition:

$$T_w > T_{sat} + \frac{dT}{dp}(P_B - P_\infty)$$

Boiling Nucleation - Boiling Inception

Slope of the vapor pressure curve from the Clausius-Clapeyron eq.

$$\boxed{\frac{dp}{dT} = \frac{h_{lv}}{(v_v - v_l)T_{sat}}}$$

h_{lv}	latent heat of vaporization
T_{sat}	saturation temperature
v_l	specific volume of the liquid
v_v	specific volume of the gas

Then, if $v_v \gg v_l$ and, since $v_v = \frac{1}{\rho_v}$

$$\frac{dp}{dT} = \frac{h_{lv}}{(v_v - v_l)T_{sat}} \implies \frac{dT}{dP} = \frac{T_{sat}}{\rho_v h_{lv}}$$

$$T_w > T_{sat} + \frac{dT}{dp}(P_B - P_\infty)$$

$$\implies T_w > T_{sat} + \frac{T_{sat}}{\rho_v h_{lv}} \frac{2\sigma}{R}$$

Critical Radius for Nucleation

If ΔT_{sat} is the value of $(T_w - T_{sat})$ at which nucleation starts, then the cavity radius given by:

$$\boxed{R = \frac{2\sigma T_{sat}}{\rho_v h_{lv} \Delta T_{sat}}}$$

For water at 1 bar, $T_{sat} = 373$ K, $\sigma = 0.059$ N/m, $h_{lv} = 2.256 \times 10^6$ J/kg, $\rho_v = 0.598$ kg/m^3, $\Delta T_{sat} \approx 5$ K.

$$R = 6.5\ \mu\text{m}$$

Typically, cavity sizes are in the micron range. If the cavity size is known, then clearly the wall superheat required to start nucleate boiling can be calculated.

Critical Radius for Nucleation

Real surfaces, of course, can contain a range of cavity sizes. As the wall superheat is increased, cavities of smaller and smaller radius are able to become active and initiate nucleation.

Maximum size of active nucleation sites on smooth metallic surfaces:
Water $\sim 5\mu\text{m}$
Organics and refrigerants $\sim 0.5\mu\text{m}$
Cryogenic fluids on aluminum or copper $\sim 0.1 - 0.3\mu\text{m}$

Heat Transfer in Nucleate Boiling

Rohsenow postulated:

- Heat flows from the surface first to the adjacent liquid, as in any single-phase convection process
- High h is a result of local agitation due to liquid flowing behind the wake of departing bubbles

Thus, it may be possible to adapt a single-phase forced convection heat transfer correlation to nucleate pool boiling, if we could specify the appropriate length and velocity scales associated with the convection process.

Heat Transfer in Nucleate Boiling

$$\mathsf{Nu} = f(\mathsf{Re}, \mathsf{Pr})$$

$$\mathsf{Nu} = \frac{hL_b}{k_l}; \quad \mathsf{Re} = \frac{\rho_v u_b L_b}{\mu_l}; \quad \mathsf{Pr} = \frac{\mu_l \; C_{Pl}}{k_l}$$

Velocity is taken as the liquid velocity towards the surface which is to supply the vapor that is being produced so:

$$u_b \sim \frac{\dot{q}''}{h_{lv}\rho_v}$$

Length scale is taken to be,

$$L_b \sim \left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{\frac{1}{2}}$$

Heat Transfer in Nucleate Boiling

$$\mathsf{Nu} = \frac{hL_b}{k_l} = \frac{h}{k_l}\left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{\frac{1}{2}}$$

$$\mathsf{Re} = \frac{\rho_v u_b L_b}{\mu_l} = \frac{\rho_v}{\mu_l}\frac{\dot{q}''}{h_{lv}\rho_v}\left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{\frac{1}{2}}$$

$$\mathsf{Pr} = \frac{\mu_l\ C_{Pl}}{k_l}$$

$$\mathsf{Nu} = \frac{1}{C_{sf}}\,\mathsf{Re}^{1-m}\,\mathsf{Pr}^{-n}$$

$$h = \frac{\dot{q}''}{T_s - T_{sat}} = \frac{\dot{q}''}{\Delta T_{sat}}$$

Rohsenow's Correlation

$$\boxed{\frac{C_{Pl}\Delta T_{sat}}{h_{lv}} = C_{sf}\left[\frac{\dot{q}''}{\mu_l\ h_{lv}}\sqrt{\frac{\sigma}{g(\rho_l-\rho_v)}}\right]^m \left[\frac{\mu_l\ C_{Pl}}{k_l}\right]^{1+n}}$$

$$\boxed{\frac{\dot{q}''}{\mu_l\ h_{lv}}\sqrt{\frac{\sigma}{g(\rho_l-\rho_v)}} = \left(\frac{1}{C_{sf}}\right)^{\frac{1}{m}}\left[\frac{C_{Pl}\Delta T_{sat}}{h_{lv}}\right]^{\frac{1}{m}} \mathrm{Pr}^{-\frac{1+n}{m}}}$$

$m = 0.33$ and $1+n = 1$ for water and 1.7 for other fluids.
Applicable only for clean surfaces

C_{sf} is the surface-fluid constant. Typically: 0.0025 and 0.015.

For a given ΔT_{sat}, $\dot{q}'' \propto C_{sf}^{-3}$.
$\because C_{sf}$ can vary by a factor of 10, $\dot{q}''$ can vary by a factor of 1000.

Rohsenow's Correlation

$$\boxed{\frac{C_{Pl}\Delta T_{sat}}{h_{lv}} = C_{sf}\left[\frac{\dot{q}''}{\mu_l\ h_{lv}}\sqrt{\frac{\sigma}{g(\rho_l-\rho_v)}}\right]^{m}\left[\frac{\mu_l\ C_{Pl}}{k_l}\right]^{1+n}}$$

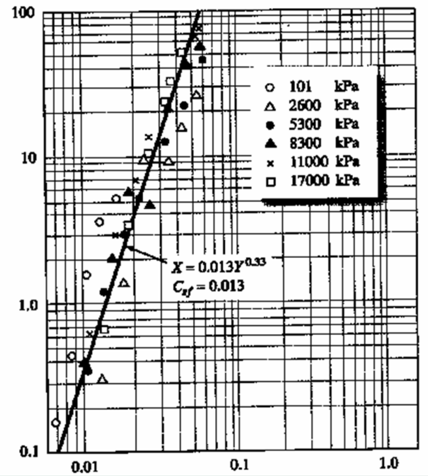
$$\boxed{\frac{\dot{q}''}{\mu_l\ h_{lv}}\sqrt{\frac{\sigma}{g(\rho_l-\rho_v)}} = \left(\frac{1}{C_{sf}}\right)^{\frac{1}{m}}\left[\frac{C_{Pl}\Delta T_{sat}}{h_{lv}}\right]^{\frac{1}{m}}\mathsf{Pr}^{-\frac{1+n}{m}}}$$

When used to estimate q'', errors can amount to $\pm 100\%$.
The errors for estimating ΔT_{sat} reduce by a factor of 3

$$\because \Delta T_e \propto (q''_s)^{1/3}$$

Rohsenow’s Method: Platinum Wire-Water

$$\left[\frac{q''}{\mu_l\, h_{lv}}\sqrt{\frac{\sigma}{g(\rho_l-\rho_v)}}\right]$$



$$\frac{C_{pl}\left(T_w - T_{sat}\right)}{h_{lv}\,\mathrm{Pr}}$$

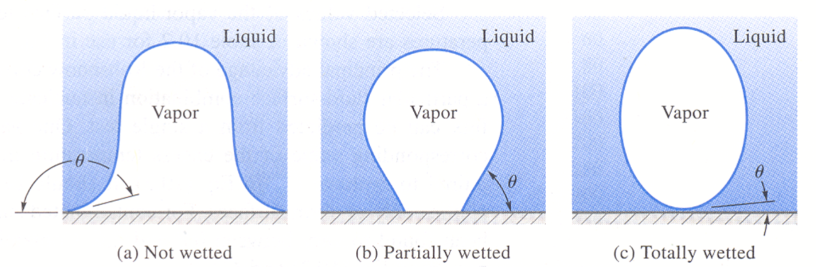
Coefficient in Rohsenow's Correlation

The most important variables affecting C_{sf} are:

- Surface roughness of the heater which determines the number of nucleation sites at a given temperature.
- Angle of contact between the bubble and heating surface, a measure of wettability of a surface with a particular fluid.

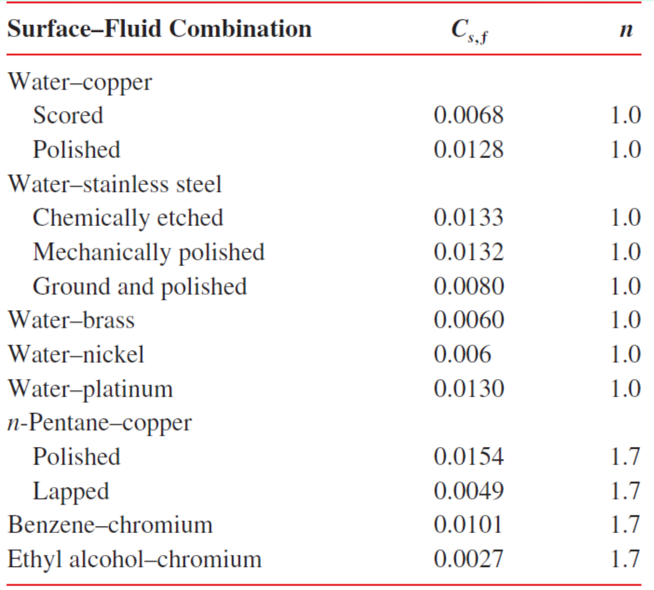
Surface is hydrophilic (wetted),if $\theta < 90^\circ$

Surface is hydrophobic (not wetted),if $\theta > 90^\circ$



(a) Not wetted (b) Partially wetted (c) Totally wetted

Coefficient in Rohsenow's Correlation



Surface–Fluid Combination	$C_{s,f}$	n
Water–copper		
Scored	0.0068	1.0
Polished	0.0128	1.0
Water–stainless steel		
Chemically etched	0.0133	1.0
Mechanically polished	0.0132	1.0
Ground and polished	0.0080	1.0
Water–brass	0.0060	1.0
Water–nickel	0.006	1.0
Water–platinum	0.0130	1.0
n-Pentane–copper		
Polished	0.0154	1.7
Lapped	0.0049	1.7
Benzene–chromium	0.0101	1.7
Ethyl alcohol–chromium	0.0027	1.7

Coefficient in Rohsenow's Correlation

- For contaminated surfaces, the exponent of Prandtl number of liquid $(1+n)$ is found to vary between 0.8 and 2.0.
- Contamination also affects the other exponent in the equation and C_{sf}
- If no data is available, then consider, $C_{sf} = 0.013$

Heat Transfer in Nucleate Boiling

Another frequently quoted Forster-Zuber correlation

$$\dot{q}'' = 0.00122\left(\frac{k_l^{0.79}C_{Pl}^{0.45}\rho_l^{0.49}}{\sigma^{0.5}\mu_l^{0.29}h_{lv}^{0.24}\rho_v^{0.24}}\right)\left[T_w - T_{sat}(P_l)\right]^{1.24}\Delta P_{sat}^{0.75}$$

ΔP_{sat} is the difference in saturation pressure corresponding to a difference in saturation temperature equal to the wall superheat $T_w - T_{sat}(P_l)$.

k_l - W/m K

C_{Pl} - kJ/kg K

ρ - kg/m^3

P - Pa

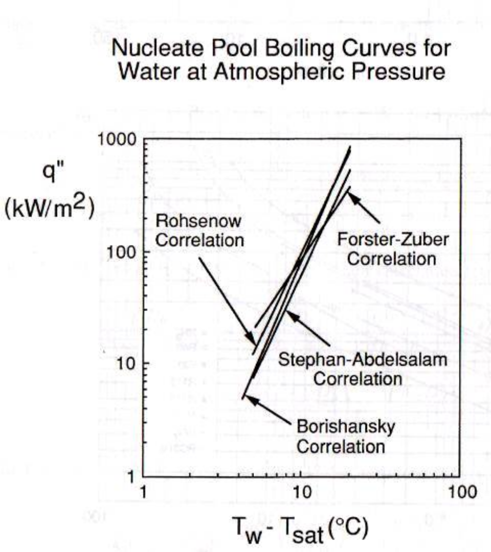
σ - N/m

μ - Ns/m^2

h_{lv} - kJ/kg

$\dot{q}''$ - kW/m^2

Heat Transfer in Nucleate Boiling



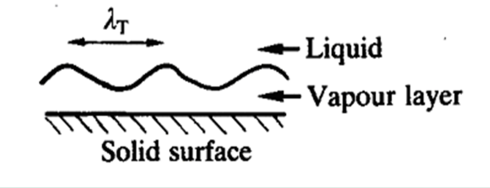
Critical Heat Flux in Nucleate Boiling

- Rohsenow's correlation is restricted to nucleate boiling
  - Does not reveal the ΔT_{sat} at which the $\dot{q}''_{max}$
  - Limiting heat flux: when nucleate boiling breaks down and an insulating vapor film forms
- For a $\dot{q}''$ controlled surface, the ΔT_{sat} rise after $\dot{q}''_{max}$ can be very large (can be > 1000 K)

Instability of Vapor Layer

The light fluid in a layer which has a heavy fluid on top of it is unstable.

The layer breaks down by the formation of waves on its surface as in the figure.



Taylor waves

Reyleigh-Taylor instability

During boiling, disturbances of all wavelengths are present, there will be some disturbances at small wavelength and long wavelength that will amplify and cause the interface to be unstable.

Rayleigh-Taylor Instability

Condition for the interface instability of a motionless liquid overlaying a motionless vapor region:

$$\alpha > \alpha_c = \left[\frac{(\rho_l - \rho_v) g}{\sigma} \right]^{\frac{1}{2}}$$

This condition is called Reyleigh-Taylor Instability

Corresponding critical wavelength:

$$\lambda_c = \frac{2\pi}{\alpha_c} = 2\pi \left[\frac{\sigma}{(\rho_l - \rho_v) g} \right]^{\frac{1}{2}}$$

The most dangerous wavelength, as they grow most rapidly,

$$\lambda_D = \sqrt{3}\ \lambda_c$$

At 1 bar:

$$\sigma = 0.058988 \text{ N/m}$$

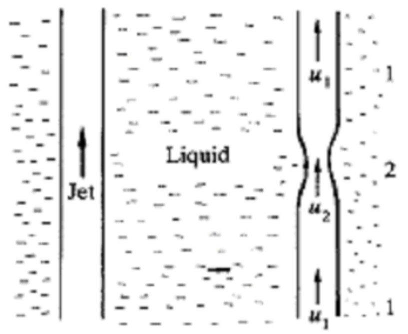
$$\rho_l = 958.63 \text{ kg/m}^3$$

$$\rho_v = 0.59034 \text{ kg/m}^3$$

$$\lambda_c = 15.7 \text{ mm}$$

$$\lambda_D = 27.2 \text{ mm}$$

Kelvin-Helmhotz Instability



A parallel sided jet is not stable.

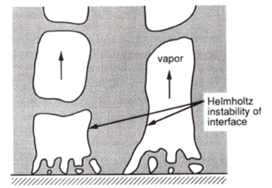
Consider the random thinning of the jet as illustrated.

By continuity, $u_2 > u_1$ and therefore, from Bernoullis equation $p_2 < p_1$.

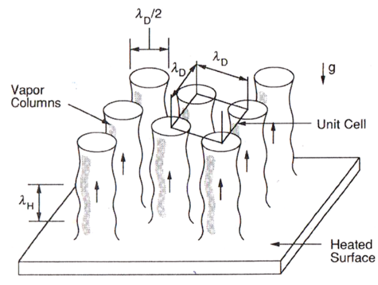
- If the jet is in equilibrium at 1, then the liquid pressure at 2 will push the 'neck' further in and disrupt the jet completely, thus breaking it up. This is a Kelvin-Helmhotz instability.
- This argument would imply that the vapor jet is always unstable, but the effects of surface tension, which has a stabilizing effect, have been neglected.

Zuber's Model

1. $\dot{q}''_{max}$ when the interface of columns - Helmholtz unstable.
2. Centerline spacing of columns $=$ most dangerous λ_D of Taylor instability.
3. Columns diameter $= \lambda_D/2$.
4. The Helmholtz unstable λ imposed on the columns $=$ Taylor wave node spacing λ_D.



Postulated Helmholtz instability CHF mechanisms



Vapor column spacing in the Zuber critical heat flux model

Zuber's Model Analysis: Consider Unit Cell

Critical Helmholtz velocity for vertical vapor and liquid flow:

$$u_c = |\bar{u}_l - \bar{u}_v| = \left(\frac{\sigma \alpha (\rho_l + \rho_v)}{\rho_l \rho_v} \right)^{\frac{1}{2}}$$

Assuming $\rho_l \gg \rho_v$ and substituting $\alpha = 2\pi/\lambda$:

$$u_c \left(\frac{2\pi\sigma}{\rho_v \lambda} \right)^{\frac{1}{2}}$$

The most dangerous wavelength from Taylor instability for liquid in contact with vapor:

$$\lambda_D = 2\pi \left[\frac{3\sigma}{(\rho_l - \rho_v) g} \right]^{\frac{1}{2}}$$

Zuber's Model Analysis: Consider Unit Cell

Critical heat flux, the rate of heat supply to the area, A_{surf}:

$$\dot{q}''_{max} A_{surf} \approx h_{lv} \rho_v u_v A_{col}$$

$$\dot{q}''_{max} = \frac{\pi}{16} h_{lv} \rho_v u_v$$

- Since the downward liquid velocity is much smaller than the upward vapor velocity, due to the large density difference between the phases, $u_c \approx u_v$.
- The Helmholtz unstable wavelength imposed on the columns is equal to the Taylor wave node spacing λ_D.

$$\dot{q}''_{max} = \frac{\pi}{16(3)^{1/4}} h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4}$$

works well for the flat horizontal plates.

Zubers Correlation for Critical Heat Flux

Zuber's correlations for flat horizontal plate:

$$\boxed{q''_{\text{max}} = 0.149 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4}}$$

works well for the flat horizontal plates.

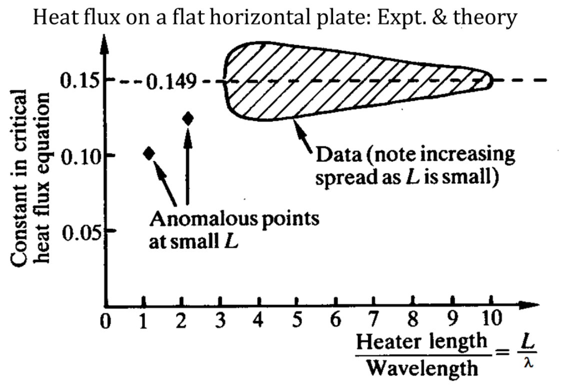
The coefficient is modified for better fit for different geometries:

$$\boxed{q''_{\text{max}} = 0.131 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4}}$$

Critical Heat Flux on a Flat Plate

$$q''_{\text{max}} = C h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4}$$

$C = 0.149$ for flat, horizontal heater



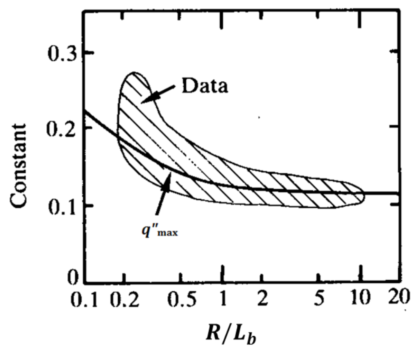
$$\lambda = 2\pi \left[\frac{\sigma}{(\rho_l - \rho_v) g} \right]^{\frac{1}{2}}$$
$$= 2\pi L_b$$

Critical Heat Flux Outside Horizontal Cylinders

$$q''_{\text{max}} = C h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4}$$

Boiling outside horizontal cylinders: expt. & theory

$$C = 0.116 + 0.3 \exp\left\{-3.44 \sqrt{\frac{R}{L_b}}\right\}$$



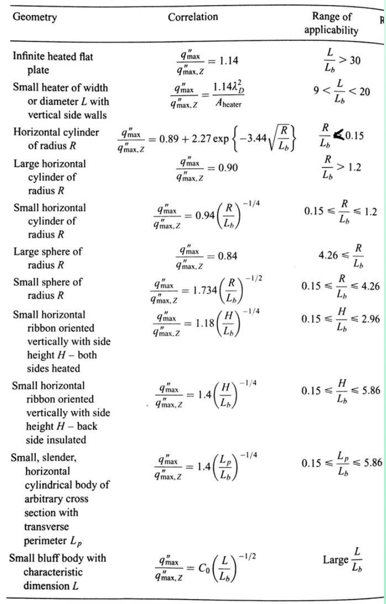
Correlations for Critical Heat Flux

$$\frac{q''_{max}}{q''_{max,z}} = f(L/L_b)$$

where $q''_{\text{max,z}} = 0.131 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2}\right]^{1/4}$ and $L_b \sim \left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{\frac{1}{2}} = \frac{\lambda_D}{2\pi\sqrt{3}}$

- The ratio L/L_b indicates the size of the heater relative to the expected spacing of the vapor columns carrying vapor away from the surface near the critical condition.
- For heaters of finite size, variation of the value of this dimensionless group (*i.e.*, the Bond number) is expected to significantly alter the CHF condition particularly if its value is near or below one.

CHF for Different Geometries of Heater

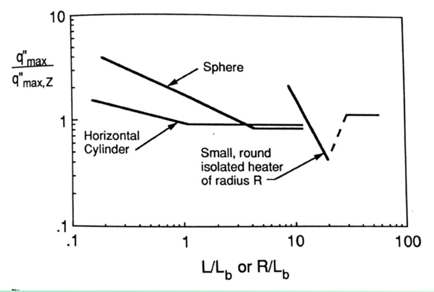


Geometry	Correlation	Range of applicability
Infinite heated flat plate	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.14$	$\frac{L}{L_b} > 30$
Small heater of width or diameter L with vertical side walls	$\frac{q''_{\max}}{q''_{\max,Z}} = \frac{1.14\lambda_D^2}{A_{\text{heater}}}$	$9 < \frac{L}{L_b} < 20$
Horizontal cylinder of radius R	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.89 + 2.27\exp\left\{-3.44\sqrt{\frac{R}{L_b}}\right\}$	$\frac{R}{L_b} \leqslant 0.15$
Large horizontal cylinder of radius R	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.90$	$\frac{R}{L_b} > 1.2$
Small horizontal cylinder of radius R	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.94\left(\frac{R}{L_b}\right)^{-1/4}$	$0.15 \leqslant \frac{R}{L_b} \leqslant 1.2$
Large sphere of radius R	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.84$	$4.26 \leqslant \frac{R}{L_b}$
Small sphere of radius R	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.734\left(\frac{R}{L_b}\right)^{-1/2}$	$0.15 \leqslant \frac{R}{L_b} \leqslant 4.26$
Small horizontal ribbon oriented vertically with side height H – both sides heated	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.18\left(\frac{H}{L_b}\right)^{-1/4}$	$0.15 \leqslant \frac{H}{L_b} \leqslant 2.96$
Small horizontal ribbon oriented vertically with side height H – back side insulated	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.4\left(\frac{H}{L_b}\right)^{-1/4}$	$0.15 \leqslant \frac{H}{L_b} \leqslant 5.86$
Small, slender, horizontal cylindrical body of arbitrary cross section with transverse perimeter L_p	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.4\left(\frac{L_p}{L_b}\right)^{-1/4}$	$0.15 \leqslant \frac{L_p}{L_b} \leqslant 5.86$
Small bluff body with characteristic dimension L	$\frac{q''_{\max}}{q''_{\max,Z}} = C_0\left(\frac{L}{L_b}\right)^{-1/2}$	Large $\frac{L}{L_b}$

$$\frac{q''_{max}}{q''_{max,z}} = f(L/L_b)$$

$$q''_{\text{max,z}} = 0.131 h_{lv}\rho_v\left[\frac{\sigma(\rho_l - \rho_v)g}{\rho_v^2}\right]^{1/4}$$

$$L_b \sim \left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{\frac{1}{2}}$$

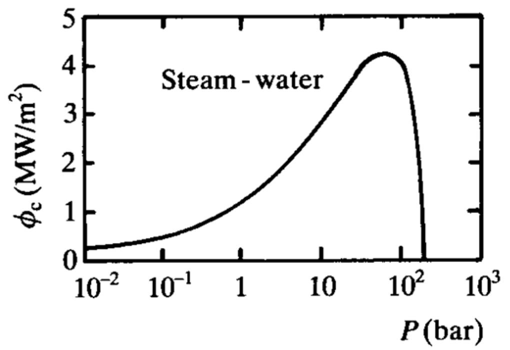


Variation of CHF with Pressure

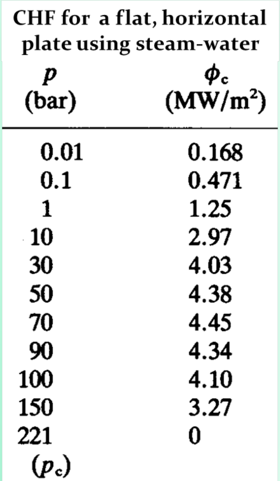
As the system pressure rises:

h_{lv} falls slowly at first & falls steeply at critical point;

ρ_v increases; σ and $(\rho_l - \rho_v)$ fall monotonically



For both pool & flow boiling, the max. CHF occurs at ~70 bar.



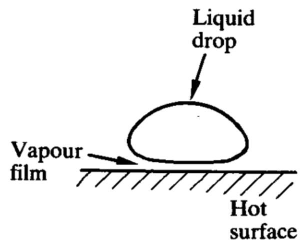
CHF for a flat, horizontal plate using steam-water

p (bar)	ϕ_c (MW/m^2)
0.01	0.168
0.1	0.471
1	1.25
10	2.97
30	4.03
50	4.38
70	4.45
90	4.34
100	4.10
150	3.27
221 (p_c)	0

Leidenfrost Temperature

Rewetting of hot surfaces: Liquid does not wet hot surface.

- Liquid is separated from the plate by a thin film of vapor so that the friction for sideways motion of the drop is very small and the heat transfer across the vapor film is poor.
- The vapor film, of course moves outwards, and fresh vapor is generated by evaporation at the underside of the drop due to heat conduction across the film and radiation from the plate to the drop.



Leidenfrost Temperature

- If the plate is allowed to cool down, it will eventually reach a temperature at which the vapor film collapses, and then very intense boiling takes place which rapidly leads to the evaporation of all the liquid.
- The surface temperature at which this sudden wetting of the plate occurs is the Leidenfrost Temperature.

$$q''_{\text{min}} = C h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

C is a non-dimensional constant which lies between 0.09 and 0.18.

$C = 0.09$ provides a better fit.

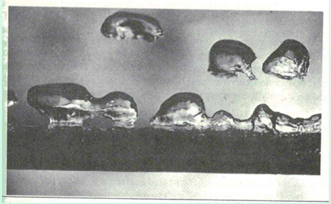
$C = 0.13$ is sometimes taken as an intermediate value

Prof. Dr. Johann G. Leidenfrost (1715-1794)

70 publications

- Father - Minister
- Started off with Theological studies
- PhD thesis, ‘’On the Harmonious Relationship of Movements in the Human Body”
- Professor at University of Duisburg
- Areas of influences:
  - Theologian
  - Physician (Private Medical practice)
  - As a Prof. taught:
    Medicine, Physics, and Chemistry

Film Boiling for Large Horizontal Surfaces



$$q''_{\text{film boiling}} = 0.425\ \Delta T_{sat} \left[\frac{\rho_v(\rho_l - \rho_v) g h'_{lv} k_v^3}{\mu_v \Delta T_{sat}} \sqrt{\frac{(\rho_l - \rho_v) g}{\sigma}} \right]^{1/4}$$

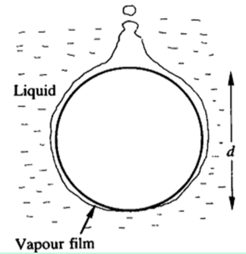
The effective latent heat of vaporization allows for the inclusion of sensible heating effects in the vapor film.

$$h'_{lv} = h_{lv} + 0.5 C_{p,v}(T_s - T_{sat})$$

Vapor properties are evaluated at the film temperature,

$$T_f = (T_s + T_{sat})/2.$$

Film Boiling



$$q''_{\text{film boiling}} = C_{\text{film}}\ \Delta T_{sat}\left[\frac{\rho_v(\rho_l-\rho_v)gh'_{lv}k_v^3}{\mu_v\Delta T_{sat}}\frac{1}{D}\right]^{1/4}$$

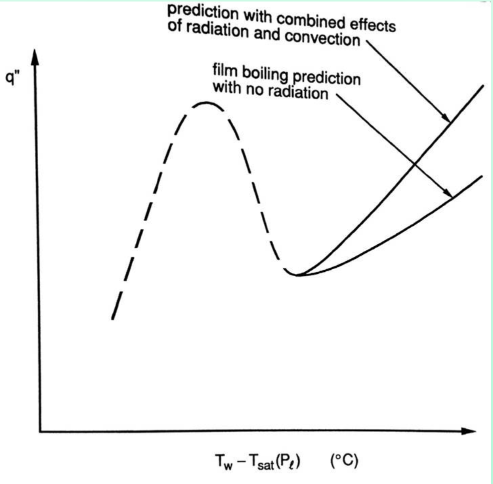
$C_{\text{film}} = 0.62$ for horizontal cylinders
$C_{\text{film}} = 0.67$ for spheres

$$h'_{lv} = h_{lv} + 0.4C_{p,v}(T_s - T_{sat})$$

To be precise, properties at $T_f = (T_s + T_{sat})/2$

Radiation Effects in Film Boiling

$$h_{\text{total}} = h_{\text{film conv}} + \frac{3}{4} h_{\text{rad}} \qquad h_{\text{rad}} = \frac{\varepsilon_s \sigma (T_s^4 - T_{sat}^4)}{T_s - T_{sat}}$$



Saturated Water-Steam on Flat Heater

$T_w = 300°\text{C}$

Physical properties of water at 1 atm and 100°C:

$$
\begin{aligned}
T_{sat} &= 100°\text{C} \\
h_{lv} &= 2257 \text{ kJ/kg} \\
k_v &= 0.0251 \text{ W/m K} \\
\mu_v &= 12.3 \times 10^{-6} \text{ Pa s} \\
\rho_v &= 0.598 \text{ kg/m}^3 \\
\rho_l &= 958 \text{ kg/m}^3 \\
\sigma &= 0.0589 \text{ N/m} \\
C_{Pv} &= 2.029 \text{ kJ/kg K}
\end{aligned}
$$

$$
\begin{aligned}
q''_{\text{max}} &= 0.149 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2}\right]^{1/4} \\
q''_{\text{min}} &= C h_{lv} \rho_v \left[\frac{g\sigma(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2}\right]^{1/4} \\
q''_{\text{film}} &= 0.425\ \Delta T_{sat} \\
&\quad \left[\frac{\rho_v(\rho_l - \rho_v) g h'_{lv} k_v^3}{\mu_v \Delta T_{sat}} \sqrt{\frac{(\rho_l - \rho_v) g}{\sigma}}\right]^{1/4}
\end{aligned}
$$

$$h'_{lv} = h_{lv} + 0.5 C_{p,v}(T_s - T_{sat})$$

Saturated Water-Steam on Flat Heater

$$\dot{q}''_{max} = 1.26 \text{ MW/m}^2$$

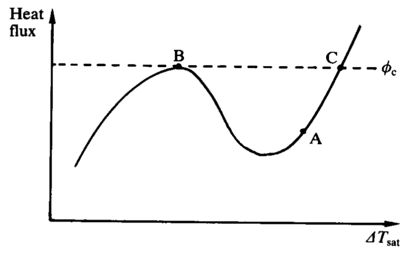
$$h_{max} = 17.5 \text{ kW/m}^2 \text{ K}$$

$$\dot{q}''_{min} = 19.03 \text{ kW/m}^2$$

$$h_{min} = 226 \text{ W/m}^2 \text{ K}$$

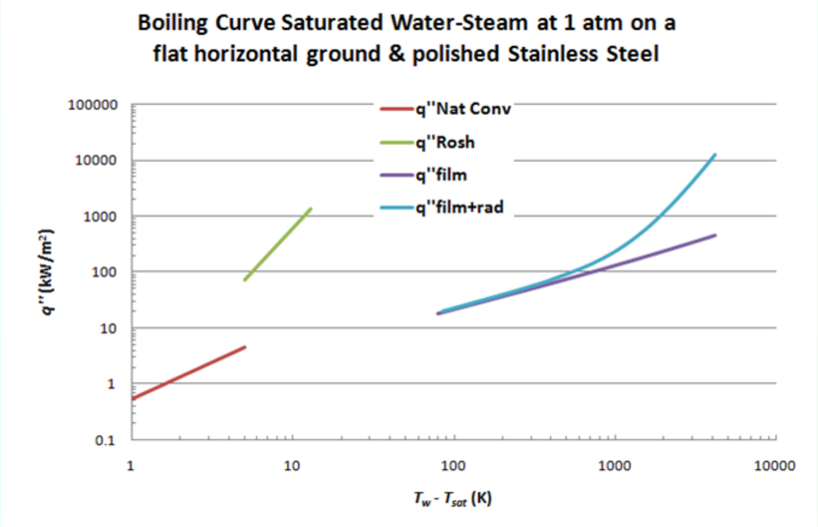
$$\dot{q}''_{film} = 39.39 \text{ kW/m}^2$$

$$h_{film} = 185 \text{ W/m}^2 \text{ K}$$

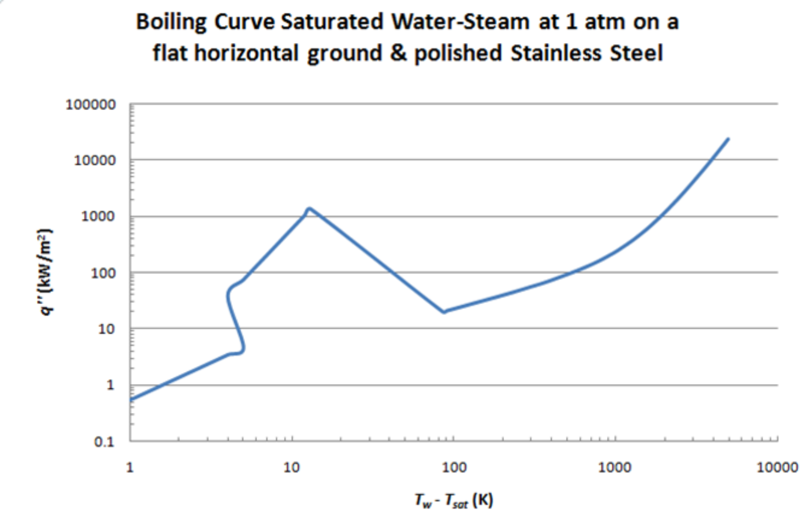


- So, although the plate is very hot, it is carrying only a fraction of the critical heat flux. So we are around point A on the boiling curve.
- The point C, which has the same heat flux as at point B, can be found.
- T_C is so high that radiative heat transfer is very important.

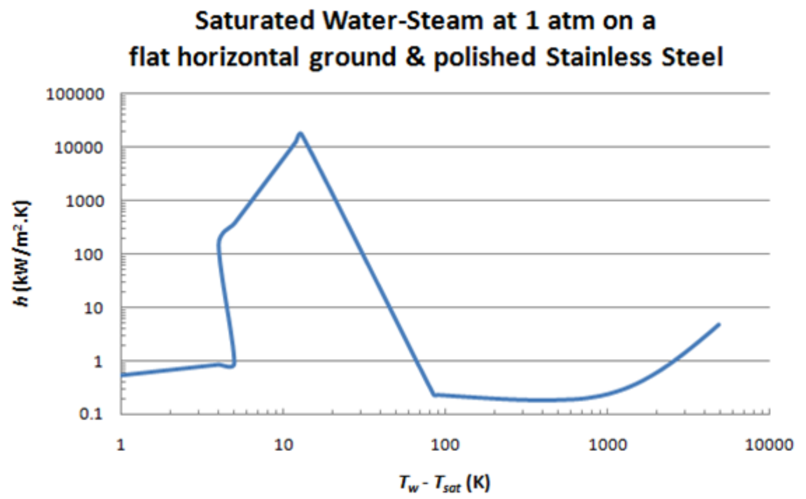
Saturated Water-Steam on Flat Heater



Saturated Water-Steam on Flat Heater



Saturated Water-Steam on Flat Heater



Effect of Liquid Subcooling

Natural convection portion will shift upward as driving $\Delta T_{sub} \uparrow$

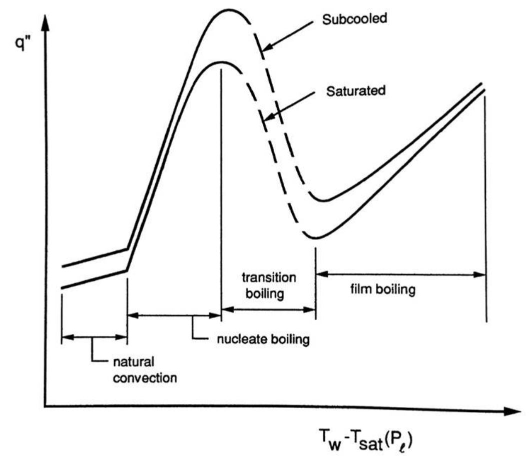
Nucleate boiling: slight influence as $\Delta T_{\mathsf{sub}} \uparrow$

A change of $\sim$300% in ΔT_{sub} produces $\sim$20% in $\Delta q''$

Maximum heat flux: strong influence

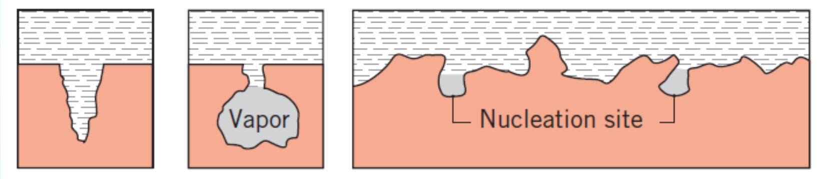
Vapor raises and condenses - easy pathway for liquid to flow towards the surface

Effect of Liquid Subcooling

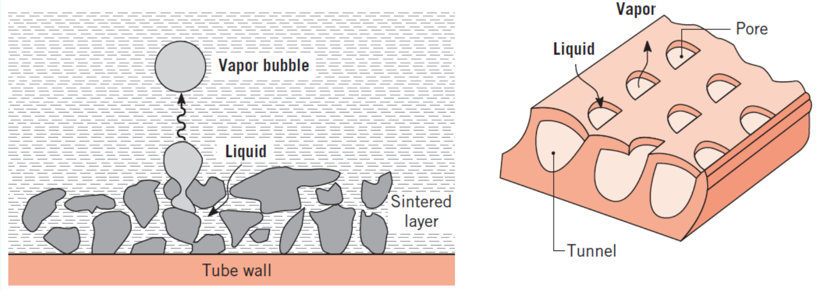


$$q''_{\text{max}} = 0.16 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4} \left[1 + .065 \left(\frac{\rho_l}{\rho_v} \right)^{0.75} \frac{C_{Pl} \Delta T_{sub}}{h_{lv}} \right]$$

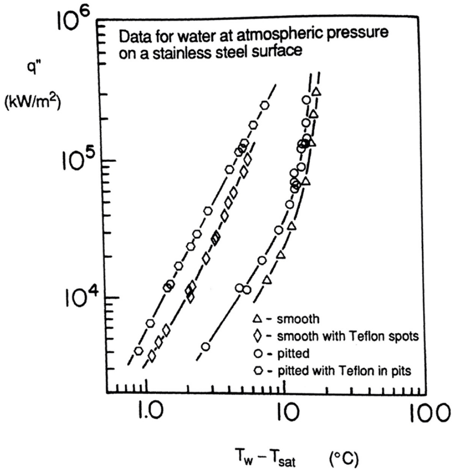
Enhancement of Heat Transfer

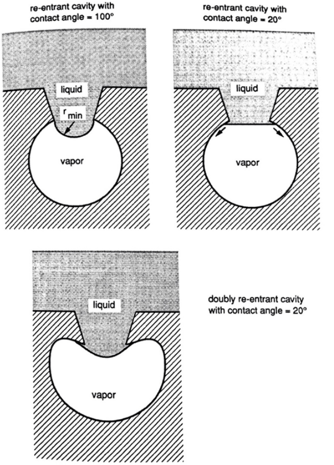


- Roughening or structuring or coating of the heating surface
- Production of artificial nucleation sites by sintering and
- Addition of gases or liquids or solids

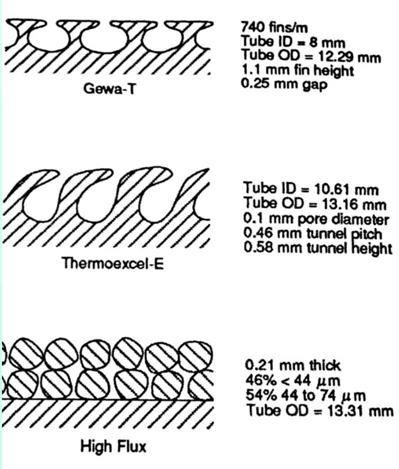


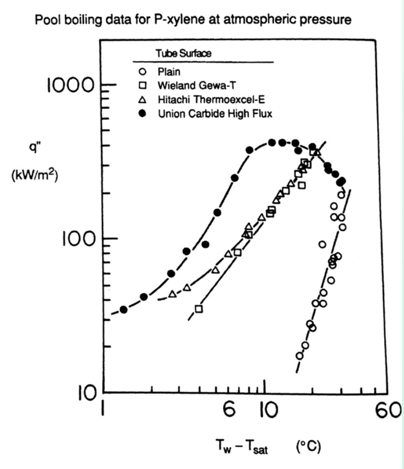
Enhancement of Heat Transfer





Enhancement of Heat Transfer





Deviation from Classical Boiling Curve

The classical boiling curves are characteristic of heat surfaces that satisfy:

1. they must be at least partially wetted by the liquid in the surrounding pool and
2. the characteristic dimension of the heat L must be large compared with the capillary length scale $L_b = \sqrt{\sigma/(\rho_l - \rho_v)g}$

If the surface does not satisfy these conditions, the resulting boiling curve can be very different from the classical curves.

Influence of Contact Angle

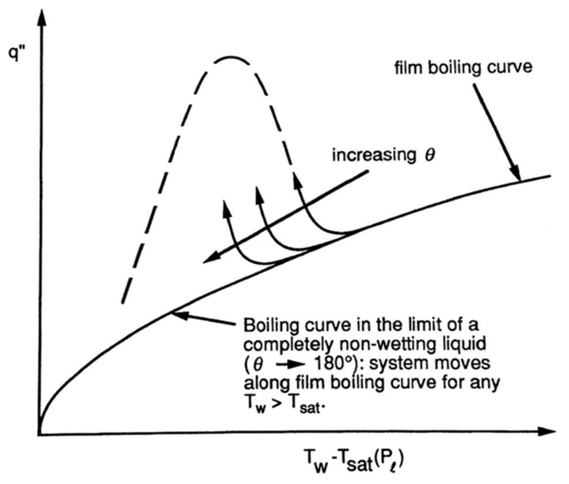
Boiling curve for a non-wetting liquid (Linear plot)

- $q'' \uparrow$ monotonically with superheat
- Eventually merges with the "classical" film boiling curve

Hydrophobic:

- Water surface coated
- Mercury on Teflon

Influence of Contact Angle

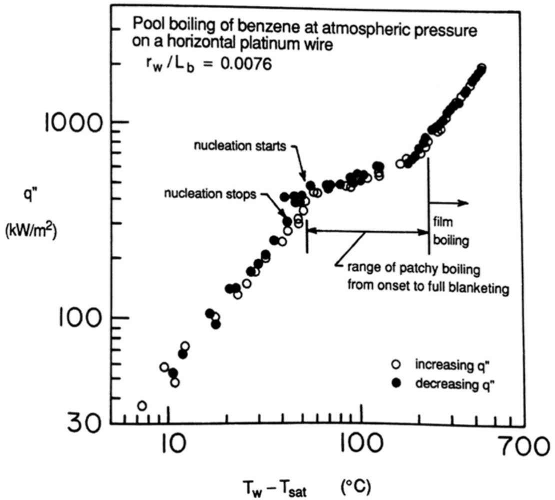


Influence of Smaller Heater

Length scale, $L_b \ll 50D_b$ (Bubble departure diameter)

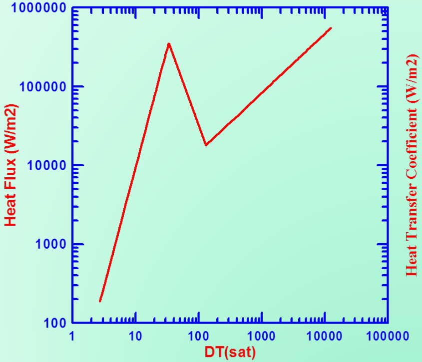
- Growing bubble completely covers the heater
- ONB initiates a film-type boiling
- No Nucleating regime
- No Transition regime
- No $\dot{q}''_{\text{max}}$

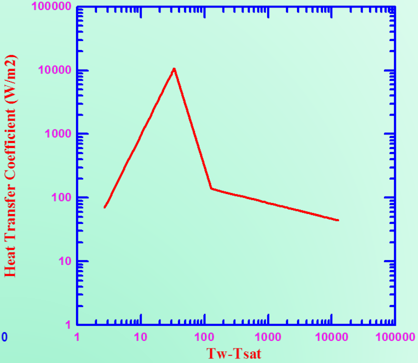
Influence of Smaller Heater



Pool Boiling curve of Benzene

Pool Boiling curve of Benzene at 1 bar & $T_{sat} = 80°\text{C}$





Problem

The bottom of a copper pan, 0.3 m in diameter, is maintained at 118°C by an electric heater. Estimate the power required to boil water in this pan. What is the evaporation rate? Estimate the critical heat flux.

Saturated water, liquid at 100°C:
$\rho_l = 957.9$ kg/m^3, $C_{P,l} = C_{P,g} = 4.217$ kJ/kg K,
$\mu_l = 279 \times 10^{-6}$ N s/m^2, $\mathrm{Pr}_l = 1.76$,
$h_{lv} = 2257$ kJ/kg, $\sigma = 0.0589$ N/m,

Saturated water, vapor at 100°C:
$\rho_v = 0.5955$ kg/m^3

Solution

Saturated water, liquid at 100°C:
$\rho_l = 957.9$ kg/m^3, $C_{P,l} = C_{P,g} = 4.217$ kJ/kg K,
$\mu_l = 279 \times 10^{-6}$ N s/m^2, Pr$_l = 1.76$,
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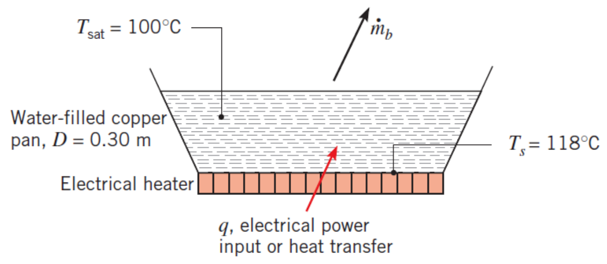
Saturated water, vapor at 100°C:
$\rho_v = 0.5955$ kg/m^3



Solution

Saturated water, liquid at $100°$C:
$\rho_l = 957.9$ kg/m^3, $C_{P,l} = C_{P,g} = 4.217$ kJ/kg K,
$\mu_l = 279 \times 10^{-6}$ N s/m^2, Pr$_l = 1.76$,
$h_{lv} = 2257$ kJ/kg, $\sigma = 0.0589$ N/m,

Saturated water, vapor at $100°$C:
$\rho_v = 0.5955$ kg/m^3

$$\frac{\dot{q}''}{\mu_l\ h_{lv}}\sqrt{\frac{\sigma}{g(\rho_l-\rho_v)}} = \left(\frac{1}{C_{sf}}\right)^{\frac{1}{m}}\left[\frac{C_{Pl}\Delta T_{sat}}{h_{lv}}\right]^{\frac{1}{m}}\text{Pr}^{-\frac{1+n}{m}}$$

$$1/m = 3, 1+n = 1, C_{sf} = 0.0068$$

$$q_s'' = 5.589 \text{ MW}$$

$$\dot{m}_{\text{evap}} = \frac{q_s}{h_{lv}} = 630 \text{ kg/h}$$

Solution

Saturated water, liquid at $100°\text{C}$:
$\rho_l = 957.9$ kg/m^3, $C_{P,l} = C_{P,g} = 4.217$ kJ/kg K,
$\mu_l = 279 \times 10^{-6}$ N s/m^2, $\text{Pr}_l = 1.76$,
$h_{lv} = 2257$ kJ/kg, $\sigma = 0.0589$ N/m,

Saturated water, vapor at $100°\text{C}$:
$\rho_v = 0.5955$ kg/m^3

$$q''_{\text{max}} = 0.149 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2} \right]^{1/4}$$

$$= 1.26 \text{ MW/m}^2$$

$$q''_{\text{min}} = 0.09 h_{lv} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

$$= 18.9 \text{ kW/m}^2$$

Solution

Saturated water, liquid at 100°C:
$\rho_l = 957.9$ kg/m^3, $C_{P,l} = C_{P,g} = 4.217$ kJ/kg K,
$\mu_l = 279 \times 10^{-6}$ N s/m^2, $\mathrm{Pr}_l = 1.76$,
$h_{lv} = 2257$ kJ/kg, $\sigma = 0.0589$ N/m,

Saturated water, vapor at 100°C:
$\rho_v = 0.5955$ kg/m^3

$$q''_{\text{max}} = 1.26 \text{ MW/m}^2 \quad \Delta T_{sat} = 10.96 \text{ K}$$

$$q''_{\text{min}} = 18.9 \text{ kW/m}^2 \quad \Delta T_{sat} = 82.72 \text{ K}$$

$$\boxed{q'' = 0.45 \text{ MW/m}^2 \quad \Delta T_{sat} = 0.45 \text{ K}}$$